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ESTIMATION OF THE MISREPORTING MODELS USING MICRO-DATA SETS DERIVED FROM THE CONSUMER EXPENDITURE SURVEY: THE GAP BETWEEN MACRO AND MICRO ECONOMIC STATISTICS ON CONSUMER DURABLES

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Abstract

In many countries, a gap between macroeconomic and microeconomic statistics is observed. To explain the gap, the present paper tests the misreporting hypothesis originally proposed by Deaton and Irish [4]. The data used for estimation involves ten clusters of consumer durables from the *Consumer Expenditure Survey* in the US. Misreporting takes place, if a household purchased goods but did not report the amount (type 1 misreporting), or it purchased goods but reported the amount incorrectly (type 2 misreporting). The variance of the measurement error in type 2 misreporting is small and is not statistically

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significant. The main source of underreporting is due to zero expenditure households that purchased goods but did not report the amount (type 1 misreporting).

1. Introduction

In many countries, there is a gap between macroeconomic and microeconomic statistics, and this gap has been the topic of various studies.[1] When values of microeconomic statistics are lower than those based on macroeconomic statistics, for the same set of items, the microeconomic statistics are considered to be underreported.

According to a seminal paper by Deaton and Irish [4], expenditures on alcoholic beverages, tobacco, and consumer durables obtained from the micro-data reported in the *Family Expenditure Survey* (FES) in the U.K. are less than those reported in the macroeconomic statistics. Based on this finding, Deaton and Irish proposed the misreporting hypothesis to explain the underreporting in microeconomic statistics.

Based on the work of Deaton and Irish [4], Maki and Nishiyama [9] tested the validity of the misreporting hypothesis by focusing on consumer durables using the micro-data sets of the *National Survey on Family Income and Expenditure* (NSFIE) in Japan. Though the idea of the misreporting hypothesis is similar to Deaton and Irish, Maki and Nishiyama replaced the p-tobit model proposed by Deaton and Irish with a p_i-tobit model that was originally proposed by Cragg [3] that referred to it as the double hurdle model.[2] The difference between the p-tobit and

[1] Houthakker and Taylor [6] are the first to investigate the gap between macro-and micro-data sets in the U.S. They report that the cross-section total, namely, the *Consumer Expenditure Survey* (CES) micro-data set, is only 93 percent of the comparable time series total, namely, the *National Income and Product Accounts in the U.S.* (NIPA) macro-data set. Slesnick [12] reports that a comparison of the estimates of aggregate expenditure by the NIPA and the CES reveals that the difference between the two data sets has been growing over time. The gap in 1961 was only 5 percent, in 1981 32 percent, and in 1989 it rose to 35 percent. Maki and Nishiyama [8], Tanner [13], Ravallion [11], and Deaton [5] also analyzed the gap between macro and micro statistics.

[2] The economic implications of Maki and Nishiyama [9] and Cragg [3] are different from each other, but the econometric specification is the same. Cragg [3] called his model the double-hurdle model.

the p_i-tobit models is that the latter assumes that the misreporting probability of each household is different according to household characteristics.

The misreporting hypothesis mainly focuses on 'zero expenditure households' in the survey. The misreporting hypothesis developed by Deaton and Irish is based on two key assumptions: First, it is assumed that there are two categories of 'zero expenditure households' in the survey: One category involves households that did not purchase some item during the survey period and correctly reported zero expenditure for the item. The other category involves households that, although they actually purchased some item during the survey period, reported zero expenditure in the survey for one reason or another. This is, the first source of misreporting (type 1 misreporting).

Second, there are 'positive expenditure households' for some items in the survey. The positive expenditure households for the items are assumed to have reported correctly their expenditures in the survey.

The extended model proposed in this paper modifies the fundamental misreporting model regarding the treatment of positive expenditure households. The second source of misreporting (type 2 misreporting) is introduced regarding positive expenditure households in the form of measurement error. That is, a household purchased some items, but reported an incorrect amount of expenditure for the items.

Section 2 introduces the models used for the estimation. The present analysis tests the misreporting hypothesis using the two different models: the p_i-tobit model and the extended model including measurement error for positive expenditure households.

Section 3 describes the CES data. We concentrate on the purchasing behavior for consumer durables to test the misreporting hypothesis specified in Section 2. The data suggest a high percentage of "zero expenditure households" for consumer durables. Thus, an application of the tobit-type qualitative response model is reasonable for analyzing demand behavior for consumer durables.

In addition, this section explains the data generating design for the estimation derived from the quarterly series of micro-data sets of CES and describes the ten categories of consumer durables used for the estimation.

Section 4 denotes the estimation method, reports the results of the empirical analysis for the two models and compares the applicability of the two models against observation data. We conclude that though the extended model is complex and general, the observation data is explained sufficiently by the p_i-tobit model.

Section 5 evaluates the results obtained by the p_i-tobit model. Section 6 explains the difference between the probability of underreporting and that of misreporting mathematically and reports the probabilities of underreporting and of misreporting. Finally, Section 7 presents some conclusions.

2. Econometric Models

2.1. The p_i-tobit model

The p_i-tobit model is specified as:

$$y_i^* = X_{1i}'\beta + u_i,\ u_i \sim N(0, \sigma^2), \tag{1}$$

$$z_i^* = X_{2i}'\gamma + v_i,\ v_i \sim N(0, 1), \tag{2}$$

$$\begin{aligned} z_i &= 1, \text{ if } z_i^* > 0 \\ &= 0, \text{ if } z_i^* \le 0, \end{aligned} \tag{3}$$

$$\begin{aligned} y_i &= y_i^*, \text{ if } y_i^* > 0 \text{ and } z_i = 1 \\ &= 0, \text{ otherwise.} \end{aligned} \tag{4}$$

The error terms, u_i and v_i, are assumed to be mutually independent.

The first equation in the model is a tobit-type demand function for consumer durables. The latent variable, y_i^*, is a function of socioeconomic

factors. The second equation reflects the household's decision about whether or not to report the purchase in the survey questionnaire. This second equation is a probit function that represents the probability that a household reported or misreported an entry of expenditure for a consumer durable.

The latent variable, z_i^*, is a function of household characteristics such as type of household, age and education. The variable of z_i takes on the value 1 with probability p_i and 0 with probability $(1 - p_i)$, where p_i and $(1 - p_i)$ are described by the distribution function as $\Phi(X_{2i}'\gamma)$ and $\Phi(-X_{2i}'\gamma)$, respectively.

The case that $z_i = 1$ and $y_i^* > 0$ indicates that a positive expenditure was reported and that, it is also correct. When $z_i = 0$ or $y_i^* \leq 0$, this indicates that a zero expenditure was reported in the survey. When $z_i = 0$ and $y_i^* > 0$, this is a case of misreporting.

The probability of reporting zero expenditure is:

$$\Phi(-X_{1i}'\beta/\sigma) + \Phi(-X_{2i}'\gamma)\Phi(X_{1i}'\beta/\sigma). \tag{5}$$

The first term of (5) corresponds to the probability that a household did not purchase consumer durables, and the second term corresponds to the probability that the household purchased consumer durables but did not report the purchase of consumer durables in the survey. This second term represents the first source of the misreporting (type 1 misreporting). The probability density of reporting positive expenditure is:

$$\Phi(X_{2i}'\gamma)\sigma^{-1}\varphi((y_i - X_{1i}'\beta)/\sigma). \tag{6}$$

The log-likelihood function of the p_i- tobit model is:

$$\ln L = \sum\nolimits_0 \ln\{1 - \Phi(X_{2i}'\gamma)\Phi(X_{1i}'\beta/\sigma)\}$$
$$- n_+ \ln\sigma + \sum\nolimits_+ \ln\Phi(X_{2i}'\gamma) + \sum\nolimits_+ \ln\varphi\{(y_i - X_{1i}'\beta)/\sigma\}. \tag{7}$$

2.2. The extended model

As an extended model of the p_i-tobit model, we applied a model including measurement error for positive expenditure. The specification of the extended model was originally introduced by Nelson [10], and Maddala [7]. The extended model is specified as:

$$y_i^* = X_{1i}'\beta + u_i,\ u_i \sim N(0, \sigma_u^2), \tag{8}$$

$$z_i^* = X_{2i}'\gamma + v_i,\ v_i \sim N(0, 1), \tag{9}$$

$$z_i = 1, \text{ if } z_i^* > 0$$

$$= 0, \text{ if } z_i^* \le 0, \tag{10}$$

$$y_i = y_i^* + w_i,\ w_i \sim N(0, \sigma_w^2), \quad \text{if } y_i^* > 0 \text{ and } z_i = 1$$

$$= 0, \text{ otherwise.} \tag{11}$$

The error terms, u_i, v_i, and w_i are assumed to be independent from each other. The probability of reporting zero expenditure is the same as before:

$$\Phi(-X_{1i}'\beta/\sigma) + \Phi(-X_{2i}'\gamma)\Phi(X_{1i}'\beta/\sigma). \tag{12}$$

The probability density of reporting positive expenditure is:

$$\Phi(X_{2i}'\gamma)\sigma^{-1}\varphi((y_i - X_{1i}'\beta)/\sigma)\Phi(y_i/\sigma_w), \tag{13}$$

where $\sigma^2 = \sigma_u^2 + \sigma_w^2$. The log-likelihood function of the extended model is:

$$\ln L = \sum\nolimits_0 \ln\{1 - \Phi(X_{2i}'\gamma)\Phi(X_{1i}'\beta/\sigma)\}$$

$$- n_+ \ln\sigma + \sum\nolimits_+ \ln\Phi(X_{2i}'\gamma) + \sum\nolimits_+ \ln\varphi\{(y_i - X_{1i}'\beta)/\sigma\}$$

$$+ \sum\nolimits_+ \ln\Phi(y_i/\sigma_w). \tag{14}$$

3. The Data Used for Estimation and Comparison

The data used for the estimation are from the public use 1994 *Consumer Expenditure Survey* (CES) quarterly interview data compiled by the Bureau of Labor Statistics, U. S. Department of Labor.

3.1. Data collection and underlying assumptions

There are basically five options that can result when using the micro-data on consumer durables, and there are three categories of the expenditure amounts, namely, (1) valid missing or blank, (2) zero, or (3) positive amounts indicated in Table 1.

Table 1. The different definition for misreporting between the BLS and the present model

	Screener	Amount	Assumed value for CES published
Case (1)	No	0	0
Case (2)	Blank	Blank (valid)	0
Case (3)	Do not know or Refusal	Blank (valid)	0
Case(4)	Yes	Positive amount	Positive amount
Case (5)	Yes	Do not know (positive amount imputed by CES office)	Positive amount

The five options for reporting expenditures in the CES are described as follows:

- **Case (1).** The household respondent reports having made no expenditure and zero expenditure is recorded.

- **Case (2).** A blank is included in the data file due to the particular type of questioning regarding the purchase of consumer durables and a valid blank enters the expenditure field.

● **Case (3).** The household respondent either says "do not know" or refuses to answer the screener question; a valid blank enters the expenditure field.

● **Case (4).** The respondent answers "yes" to the screener question regarding whether a purchase was made and reports an amount of expenditure.

● **Case (5).** The respondent reports making a purchase, but fails to report the expenditure. The BLS imputes a positive amount of expenditure.

The five options for response are based on how expenditure data are collected. In the survey, households are usually asked two questions about items of expenditures: one is a screener regarding whether or not, there was a purchase and the other is a question regarding the amount that a respondent paid for an item.

There are two types of screener questions; one involves a straightforward question about a particular item, for example, "Did you buy any magazines?" If the household reports "no", a "valid missing" is recorded in the data for the expenditure. For publication purposes, the "valid missing" is converted to a zero.

The other type of screener question covers broad categories of items such as 'Appliances and household equipment.' In this type of screener, the respondent picks items purchased from the "laundry list" indicated in the 'Appliances and household equipment' category covering forty items ranging from 'Small electric kitchen appliances' to 'Telephones and accessories.' If the respondent had an expense, say, for a toaster, a screener record is created in the category of 'Small electric kitchen appliances' in which the toaster is included. On the other hand, if the respondent had no purchase, say, for 'Telephone accessories' in the sample period, no screener record is created for the category of 'Telephones and accessories.'

If the answer to the screener is "do not know", or there is a refusal to answer the screener question, the screener record is specified as "do not know or refusal" and the amount in the expenditure field for the item is specified as a "valid blank" in the data. For the publication of means and aggregates, "valid blanks" are recorded to zero.

The fourth option is when the household respondent notes that a purchase was made and reports an amount of expenditure. The fifth option is, if a respondent answered "yes" to the purchase screener, but answered "do not know" or refused to provide the expenditure, then an amount of expenditure is imputed by the BLS.

In the p_i- tobit model, misreporting is considered to take place in the case of zero expenditure households, specifically, in cases (1), (2) and (3). On the other hand, in the extended model, misreporting takes place both for zero and positive expenditures.

3.2. Data coverage and structure

The 1994 CES public use file includes a full accounting of expenditures for December 1993-December 1994 and a partial account of expenditures for October-November 1993 and January-February 1995. Data were collected from January 1994 through March 1995. During this time period, some households had one interview, some had two interviews, some had three interviews, and some had four interviews. The number of interviews is related to the quarter, in which the households entered the rotating CES sample and to the households participation in the series of quarterly interviews for which they were selected.

For the present analysis, we start with arranging the identification number of households to group households into seven sets of households. We made seven sets mainly because we analyze purchases of consumer durables for the same households during the period of 1994. The definition of a consumer durable is that, it can be used for over one year due to its durability.

In Table 2, the total number of households used for estimating the model is 7,108. There are seven sets of households with varying reference periods depending on the number of quarters that the household participates in the interview during the 1994 reference period. The first panel of Table 2 shows the rearrangement of households from the first to the seventh set.

Table 2. Rearrangement of households

(a) Seven sets

Collection Period	Interview Number			
	2nd	3rd	4th	5th
Q1	set 4	set 3	set 2	set1
Q2	set 5	set 4	set 3	set 2
Q3	set 6	set 5	set 4	set 3
Q4	set 7	set 6	set 5	set 4

(b) Number of households in the seven sets

Stream	Number of sample observations	Collection Period
Set 1	1,172	1994 Q1
Set 2	1,073	1994 Q1, Q2
Set 3	953	1994 Q1, Q2, Q3
Set 4	891	1994 Q1, Q2, Q3, Q4
Set 5	944	1994 Q2, Q3, Q4
Set 6	991	1994 Q3, Q4
Set 7	1,084	1994 Q4
total	7,108	

Here, the case of set 1 indicated in Table 2(a) is explained. The households included in set 1 had the final or fifth interview in the first quarter of 1994, and thus, they were not part of the sample after that

quarter.[3] They reported their expenditure behavior for the last quarter in 1993 and up to the first two months of 1994. They could also have reported their expenditures for the second and third quarters of 1993 as well, but those data are out of the scope of the present study. Only their expenditures for December 1993-February 1994 are included in the 1994 public use data. The number of households classified in set 1 is 1,172 as indicated in Table 2(b).

In the case of set 4, households had a second interview in the first quarter of 1994, a third interview in the second quarter of 1994, a fourth interview in the third quarter of 1994, and the fifth and final interview in the fourth quarter of 1994. The number of households included in set 4 is 891. Finally, in the case of set 7, households had the second interview in the fourth quarter of 1994 and would have had consecutive interviews in the first three quarters of 1995.

3.3. Data on consumer durables

Consumer durables are divided into ten clusters of items using the classifications for Personal Consumption Expenditures (PCE) by type of expenditure (Table 2.5.5) included as a part of the *National Income and Product Accounts* published by the Bureau of Economic Analysis for 1994 (BEA [2]) to compare microeconomic statistics with macroeconomic statistics. Each cluster is identified by an item number. It corresponds to the line number of the durable as noted in PCE Table 2.5.5. The ten clusters are: Jewellery and watches (item 18), Furniture, including mattresses and bed springs (item 29), Kitchen and other household appliances (item 30), China, glassware, tableware, and utensils (item 31), Other durable house furnishings (item 32), Ophthalmic products and orthopedic appliances (item 46), Tires, tubes, accessories, and other parts (item 73), Books and maps (item 87), Wheel goods, sports and photographic equipment, pleasure boats and aircraft (item 90), and Video

[3] The first interview of the survey is for bounding and the expenditures collected are not used for this study or by BLS for publication.

and studio products, computing equipment, and musical instruments (item 91).[4]

3.4. Independent and dependent variables

Table 3 shows a set of independent variables specified in the models. It identifies two types of variables with the continuous variables listed first, and then the dummy variables. Table 3 also depicts the mean, standard deviation, minimum and maximum of each variable.

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[4] We excluded from the present report for the estimation results of the vehicle related three categories; namely, New autos (item 70), Net purchases of used autos (item 71) and Other motor vehicles (item 72). There are a couple of reasons for these omissions. The first is that there is a problem for the CES (corresponded to microeconomic statistics)-PCE (corresponded to macroeconomic statistics) comparison for vehicle purchases, namely, the ratio between CES and PCE exceeds unity. The ratio between CES and PCE in 1994 is 1.14. It is not surprising that the CES to PCE ratio is greater than 1.0 since the CES aggregate includes expenditures for person-to-person sales, while the PCE does not. This difference is most likely due to definitional difference and not over-reporting. The CES-PCE comparisons that are presented in the CES publications are based on the assumption that annual expenditures for the category can be determined by multiplying quarterly expenditures by four. In the aggregate, this method would be fine. However, we cannot follow this procedure as we base our model on individual, rather than aggregate, household expenditures and characteristics. The second is due to the different estimation methods for vehicle related categories between CES and PCE. CES uses a family expenditure survey method focusing on households, while PCE uses a commodity-flow method mainly focusing on retail statistics. In the footnote of the Table in *Consumer Expenditure Survey*, the BLS explains the difference between the family expenditure survey method and the commodity-flow method regarding vehicle purchases as follows: PCE estimates are derived, using estimates of dealer margin (a concept which cannot be matched to CES) and wholesale value of net transactions between persons and government, foreigners, and non-dealer businesses. CES data on vehicle purchases and trade-ins were combined to approximate the total value of new vehicle purchases. CES data on used vehicle purchases, trade-ins, sales, and losses were combined to approximate the values of net transactions of used vehicles. The most difficult issue in interpreting retail statistics is to separate automobile purchases between a household's family and business uses. The third is due to our preliminary estimation results of the model regarding vehicle related categories. Though, we estimated the econometric models for the above categories, the present model is too simple to get reasonable estimates. For the vehicle related categories, a more complex model than the present one is necessary.

Table 3. Statistics of the independent variables

	Mean	Std. dev.	Minimum	Maximum
Age of reference person	48.74	17.25	16	90
Number of members	2.62	1.51	1	13
Income before tax	35968.0	30900.0	2	293000.0
Number of weeks worked	16.8	23.3	0	52
Number of rooms	5.76	1.96	1	18
Total expenditure last quarter	17864.2	14664.5	716.6	184354.0
Urban and rural				
urban	.888	.316	0	1
rural	.112	.316	0	1
Housing tenure				
owner	.669	.470	0	1
rented	.310	.462	0	1
other	.021	.142	0	1
Education				
less than high	.194	.395	0	1
high school	.552	.497	0	1
college	.140	.346	0	1
graduate	.114	.317	0	1
Family type				
husband and wife	.207	.405	0	1
H/W own child/children	.297	.456	0	1
others	.044	.204	0	1
one parent	.062	.242	0	1
single	.265	.441	0	1
other	.125	.330	0	1
Employer status				
no employment	.253	.434	0	1
private	.535	.498	0	1
federal	.130	.336	0	1
self-employed	.082	.274	0	1

Table 4 presents statistics regarding dependent variables and the number of zero expenditure households (cases (1), (2), and (3) in Table 1) for ten clusters of consumer durables. We find that the rate of zero expenditure households for various commodity groupings ranges from 53 percent up to 81 percent.

Table 4. Statistics of dependent variables

	Mean	Std. dev.	Min.	Max.	No. of zero expenditure
Jewellery and watches					
DEP18	76.6	357.0	0	10500.0	4646 (65.3%)
Furniture, including Mattresses, and bedsprings					
DEP29	181.1	652.0	0	17700.0	5015 (70.5%)
Kitchen and other household appliances					
DEP30	74.3	247.1	0	5322.0	4769 (66.9%)
China, glassware, tableware, and utensils					
DEP31	20.1	77.9	0	1902.0	5383 (75.7%)
Other durable household furnishings					
DEP32	115.6	369.6	0	10073.0	3817 (53.7%)
Ophthalmic products and orthopedic appliances					
DEP46	31.4	89.4	0	1512.0	5776 (81.2%)
Tires, tubes, accessories, and other parts					
DEP73	35.4	143.7	0	5150.0	5285 (74.3%)
Books and maps					
DEP87	15.9	59.5	0	1500.0	5722 (80.5%)
Wheel goods, sports and photographic equipment, boats, and pleasure aircraft					
DEP90	489.9	6266.5	0	419626.0	4846 (68.1%)
Video and audio products, computing equipment, and musical instruments					
DEP91	137.3	359.9	0	6000.0	4076 (57.3%)

4. The Estimation Results

4.1. The p_i-tobit model

The estimation equation of the p_i-tobit model for the item is:

$$\ln L = \sum_0 \ln\{1 - \Phi(X_{2i}'\gamma)\Phi(X_{1i}'\beta/\sigma)\} - n_+ \ln\sigma + \sum_+ \ln\Phi(X_{2i}'\gamma) + \sum_+ \ln\varphi\{(y_i - X_{1i}'\beta)/\sigma\}, \quad (15)$$

where y_i is the amount of expenditure, X_{1i} and X_{2i} are the matrix of independent variables, respectively. The β's, γ's, and σ are parameters to be estimated.

Socio-economic variables for determining the purchasing behavior of consumer durables in Equation (1) of Subsection 2.1 are selected such as housing tenure, pre-tax income, number of family members, family type, number of weeks worked, and number of rooms. Variables concerning household characteristics in Equation (2) are selected such as age, urban/rural, education, employer status, and total expenditure last quarter.

The maximum likelihood estimators are obtained, and are indicated in the first panel of Table 5 for Jewellery and watches (item 18), and in the second panel for Wheel goods, sports and photographic equipment, pleasure boats and aircraft (item 90).[5]

[5] In this paper, we showed the estimation result only in the cases of Jewellery and watches (item 18), and Wheel goods, sports and photographic equipment, boats and pleasure aircraft (item 90) in Table 5 because of space limitations. In the case of Tires, tubes, accessories, and other parts (item 73), convergent parameters of the model could not be obtained from the common set of independent variables. In item 73, the variable family type is excluded from Equation (1) and the variable employer status from Equation (2) to obtain convergence.

Table 5. Maximum likelihood estimator: Jewellery and watches (item 18) and Wheel goods, sports and photographic equipment, pleasure boats and aircraft (item 90)

(1) Jewellery and watches (item 18).

(1.1) Tobit function:

Variables	p_i- tobit model	Extended model
Constant (β_0)	–638.8 (11.6)	–636.7 (11.3)
Housing tenure		
rented (β_1)	–12.3 (0.4)	–12.3 (0.4)
other (β_2)	–212.1 (2.1)	–212.1 (2.5)
Income before tax		
(β_{10})	.0057 (9.6)	.0057 (7.0)
Income before tax squared		
(10^{-8})(β_3)	–.622 (2.4)	–.622 (1.2)
Number of members		
(β_4)	–6.03 (0.5)	–6.03 (0.5)
Family type		
own child (β_5)	36.7 (1.0)	36.7 (1.0)
other H/W (β_6)	115.7 (2.5)	115.7 (1.9)
1 parent (β_7)	–28.9 (0.4)	–28.9 (0.5)
single (β_8)	–91.0 (2.3)	–91.0 (2.4)
other (β_9)	–21.0 (0.5)	–21.0 (0.5)
No. of weeks worked		
(β_{11})	–.530 (0.9)	–.530 (0.9)
No. of rooms		
(β_{12})	21.4 (4.0)	21.4 (3.5)
Standard error		
σ	653.2(241.9)	653.1 (66.4)
σ_w		.2395 (0.1)

(1.2) Probit function

Variables	p_i- tobit model	Extended model
Constant (γ_0)	1.613 (1.4)	1.613 (1.1)
Age (γ_1)	–.039 (2.4)	–.039 (2.1)
Urban/Rural		
rural (γ_2)	–.592 (1.9)	–.592 (1.5)
Education		
high school (γ_3)	.827 (2.7)	.873 (3.2)
college (γ_4)	.684 (1.0)	.684 (1.2)
graduate (γ_5)	1.39 (1. 0)	1.39 (1.0)
Employer		
private (γ_6)	.463 (0.9)	.463 (0.8)
federal (γ_7)	.312 (0.4)	.312 (0.2)
self-employed (γ_8)	.589 (0.6)	.589 (0.7)
Total expenditure last quarter		
(γ_9)	.00015 (3.4)	.00015 (2.9)

Note: The figures in parentheses denote asymptotic *t*-ratio.

(2) Wheel goods, sports and photographic equipment, pleasure boats and aircraft (item 90).

(2.1) Tobit function:

Variables	p_i- tobit model	Extended model
Constant (β_0)	–14114.3 (10.1)	–14100.1 (13.5)
Housing tenure		
rented (β_1)	–556.4 (0.8)	–556.6 (1.1)
other (β_2)	1125.2 (0.8)	1124.6 (0.8)
Income before tax		
(β_{10})	.133 (7.4)	.133 (8.7)
Income before tax squared		
(10^{-8})(β_3)	–49.7 (4.8)	–49.6 (5.1)

Number of members		
(β_4)	260.2 (0.8)	259.9 (1.2)
Family type		
own chid (β_5)	392.1 (0.4)	392.6 (0.6)
other H/W (β_6)	−2031.6 (1.2)	−2029.0 (1.7)
1 parent (β_7)	−508.1 (0.3)	−507.5 (0.5)
single (β_8)	−1010.4 (1.0)	−1009.3 (1.4)
other (β_9)	−1151.4 (1.1)	−1149.9 (1.4)
No. of weeks worked		
(β_{11})	27.9 (1.9)	27.9 (2.6)
No. of rooms		
(β_{12})	255.7 (1.7)	255.3 (2.2)
Standard error		
σ	11454.0 (571.4)	11443.4 (66.0)
σ_w		.2703 (0.1)

(2.2) Probit function

Variables	p_i- tobit model	Extended model
Constant (γ_0)	3.55 (3.1)	3.55 (3.3)
Age (γ_1)	−.067 (4.2)	−.067 (5.2)
Urban/Rural		
rural (γ_2)	.423 (1.0)	.423 (1.0)
Education		
high school (γ_3)	.873 (3.2)	.873 (3.2)
college (γ_4)	1.02 (1.9)	1.02 (2.1)
graduate (γ_5)	2.70 (0.7)	2.70 (1.1)
Employer		
private (γ_6)	.106 (0.2)	.106 (0.2)
federal (γ_7)	−.851 (1.4)	−.851 (1.7)

self-employed (γ_8)	–.128 (0.2)	–.128 (0.2)
Total expenditure last quarter (γ_9)	.000099 (3.0)	.000099 (3.1)

Note: The figures in parentheses denote asymptotic *t*-ratio.

4.2. The extended model

The estimating equation of the extended model for the item is:

$$\ln L = \sum_0 \ln\{1 - \Phi(X_{2i}'\gamma)\Phi(X_{1i}'\beta / \sigma)\} - n_+ \ln\sigma + \sum_+ \ln\Phi(X_{2i}'\gamma) + \sum_+ \ln\varphi\{(y_i - X_{1i}'\beta)/\sigma\} + \sum_+ \ln\Phi(y_i / \sigma_w), \quad (16)$$

where y_i is the amount of expenditure, X_{1i} and X_{2i} are the matrix of independent variables, respectively, and $\sigma^2 = {\sigma_u}^2 + {\sigma_w}^2$. The β's, γ's, σ_u, and σ_w are parameters to be estimated.

The maximum likelihood estimators are obtained, and are indicated in the first panel of Table 5 for Jewellery and watches (item 18), and in the second panel for Wheel goods, sports and photographic equipment, pleasure boats and aircraft (item 90).

Although, we have introduced the error component in the extended model, the variance of σ_w in the model is not significant statistically as indicated in Table 5. Table 6 indicates changes in $\sigma\,(\sigma = \sqrt{({\sigma_u}^2 + {\sigma_w}^2)})$ and σ_w in the extended model from σ in the p_i-tobit model. The value of σ for a given category is almost entirely the same as the σ in the p_i-tobit model.

Table 6. Changes in σ and σ_w in the extended model from σ in the p_i-tobit model

	Extended model		p_i-tobit model
	σ	σ_w	σ
Jewellery and watches (item 18)	653.1 (66.4)	.2395 (0.1)	653.2 (241.9)
Furniture, including mattresses and bed springs (item 29)	1385.8 (59.0)	.5464 (0.2)	1386.2 (166.7)
Kitchen and other household appliances (item 30)	-		498.4 (177.6)
China, glassware, tableware, and utensils (item 31)	188.0 (52.7)	.2350 (0.2)	188.0 (153.2)
Other durable house furnishings (item 32)	576.4 (76.7)	.2551 (0.2)	577.2 (291.5)
Ophthalmic products and orthopedic appliances (item 46)	-		280.6 (51.7)
Tires, tubes, accessories, and other parts (item 73)	335.9 (54.7)	.2592 (0.1)	336.8 (225.9)
Books and maps (item 87)	-		170.5 (115.0)
Wheel goods, sports and photographic equipment, pleasure boats and aircraft (item 90)	11443.4(66.0)	.2703 (0.1)	11454.0 (571.9)
Video and studio products, computing equipment, and musical instruments (item 91)	603.1 (72.2)	.2490 (0.2)	603.1 (183.6)

Note: In the extended model $\sigma^2 = {\sigma_u}^2 + {\sigma_w}^2$ and in the p_i-tobit model $\sigma^2 = {\sigma_u}^2$.

The σ_w is small and it is not significant statistically. This suggests that the σ_w does not play an important role in the model. We conclude from the present analysis that positive expenditure households reported the amount of consumer durables correctly from a statistical point of view. Comparing the two econometric models, we selected the p_i-tobit model.

5. Evaluation of the Results

There are several findings based on the estimation results by the p_i-tobit model. At first, we focus on the tobit-type demand function of Equation (1) for each item (see Table 5 and the β's). Regarding housing tenure, there are several categories whose demand behavior is different between owner housing tenure and rental housing tenure. The owner housing tenure is set as a benchmark in the estimation. The parameter of rental housing tenure is negative and statistically significant in four clusters of items. This indicates, other conditions being equal, that demand for such items is less among consumers of rental housing tenure compared to that for owner housing tenure.

We included two variables regarding income, namely, income and income squared. The parameter of income is positive and significant for all items, while that of the square of income is negative and significant for all items.

Family type appears to be an important factor affecting the level of demand. We classified family type in six categories: namely, (i) husband-and-wife only, (ii) husband-and-wife with children, (iii) all other cases of households with husband-and-wife, (iv) single-parent households, (v) single-person households, and (vi) others. In the model, households with husband-and-wife only serve as a benchmark in the estimation.

There is a big difference in the consumption of consumer durables between husband-and-wife only households and single-person households. This difference is evident because almost all of the dummy variables for single people are significant and negative for two items. This indicates that demand for consumer durables is less in single-person households compared to that of husband-and-wife only households. The parameter of single people in Books and maps (item 87) is positive and significant, indicating that single person households purchase more books and maps than husband-and-wife only households.

Space is one of the important factors, when considering purchasing consumer durables. In the set of independent variables, the number of rooms variable is a variable for "space". The parameter for the number of rooms is significant for six items.

We then consider the parameters of the probit functions, γ, related to the probability of misreporting indicated by $\Phi(-X_{2i}'\gamma)\Phi(X_{1i}'\beta/\sigma)$. The parameter of age of the reference person is negative and significant for all clusters of items except Kitchen and other household appliances (item 30), and Books and maps (item 87), indicating that the probability of misreporting increases according to the increase in the age of the reference person.

When the parameter of the rural dummy variable is negative, we conclude that, other conditions being equal, the probability of misreporting for rural households is high compared to urban households and *vice versa*. The parameter of the rural dummy is significant in Tires, tubes, accessories, and other parts (item 73). Regarding the above category, the rural dummy is positive, indicating that the probability of misreporting for the item is lower in rural households than in urban households, when $\Phi(X_{1i}'\beta/\sigma)$ is the same.

Education affects the probability of misreporting for households. The education dummy is classified into four categories, namely, (i) less than high school graduate, (ii) less than college graduate, (iii) college graduate, and (iv) graduate school graduate. The base dummy in the estimation is the category less than high school graduate. Almost, all the education dummies for high school education and higher are positive and significant, indicating that the probability of misreporting becomes less with increases in the level of education.

We consider the variable of total expenditure last quarter as a proxy for psychological comfort in the sense that the greater the total expenditures, holding all other variables constant, the more time the household spent with the interviewer to provide more complete

responses. It is hypothesized that when a household is experiencing various difficulties, it is less likely to spend enough time to report the details of expenditures to the survey interviewer. The parameter for total expenditures is positive and significant for all the categories. So, this means as total expenditures increase the probability of misreporting decreases.

6. Probabilities of Underreporting and Misreporting

Here, we clarify the difference between the probabilities of underreporting and misreporting for a household derived from the model. The first denotes the probability of misreporting for a household and the second denotes the probability of underreporting for a household:

(a) The probability of misreporting for a household is expressed as:

$$\begin{aligned} \text{Prob}^1 &= \Pr(y_i^* > 0,\ z_i^* \le 0) \\ &= \Phi(-X_{2i}'\gamma)\Phi(X_{1i}'\beta/\sigma). \end{aligned} \tag{17}$$

(b) The probability of underreporting for a household is expressed as:

$$\begin{aligned} \text{Prob}^2 &= (0 \cdot \Pr(y_i^* \le 0) + 0 \cdot \Pr(y_i^* > 0,\ z_i^* \le 0)) + (x \cdot \Pr(y_i^* > 0,\ z_i^* > 0)) \\ &\quad / (0 \cdot \Pr(y_i^* \le 0) + x \cdot \Pr(y_i^* > 0)) \\ &= \Pr(y_i^* > 0,\ z_i^* > 0) / \Pr(y_i^* > 0) \\ &= \Pr(z_i^* > 0 \mid y_i^* > 0) \\ &= \Phi(X_{2i}'\gamma)\Phi(X_{1i}'\beta/\sigma) / \Phi(X_{1i}'\beta/\sigma) \\ &= \Phi(X_{2i}'\gamma), \end{aligned} \tag{18}$$

where x represents the amount of expenditure.

In a subsequent section, we use the probability of underreporting across households and the underreporting rate in relation to the probability of misreporting across households. The definitions of the three concepts are as follows:

(1) The average probability of underreporting *across* households is defined as:

$$\text{Prob}^3 = (1/n)\sum_i \Pr(z_i^* > 0 \mid y_i^* > 0) = (1/n)\sum_i \Phi(X_{2i}'\gamma). \qquad (19)$$

When Prob^3 is less than 1.0, underreporting results. When the probability is equal to 1.0, there is no underreporting in the expenditure data.

(2) Underreporting rate defined in terms of CES and PCE aggregate expenditures.

Prob^4: ratio of CES to PCE aggregate expenditures published by the BLS.

The data used for the underreporting rates (Prob^4) are ratios of CES to PCE aggregate expenditures for selected expenditure categories for 1994. Ratios of CES to PCE aggregate expenditures are published biennially in regular reports. The data for 1994 were published in BLS Report 935, *Consumer Expenditure Survey,* 1996-97 (BLS [1]).

(3) The average probability of misreporting *across* households is defined as:

$$\text{Prob}^5 = (1/n)\sum_i \Pr(y_i^* > 0,\ z_i^* \le 0)$$

$$= (1/n)\sum_i \Phi(-X_{2i}'\gamma)\Phi(X_{1i}'\beta/\sigma). \qquad (20)$$

When both Prob^3 and Prob^4 are the same as unity, there is no underreporting or misreporting, and thus no difference between the microeconomic and macroeconomic statistics. On the other hand, when both are the same, but not at unity, there is a gap between microeconomic and macroeconomic statistics. This gap is explained fully by the misreporting hypothesis. For example, let us say that both Prob^3 and Prob^4 are 0.93 indicating that the CES microeconomic data are underreported. The underreporting in the microeconomic data is due to misreporting; the impact of this misreporting is reflected in the underreporting rate based on the CES to PCE aggregates. Finally, when both Prob^3 and Prob^4 are

different and not at unity, the difference or gap in microeconomic and macroeconomic data may be explained by other factors in addition to the misreporting hypothesis.

The average of the probabilities of underreporting by households, defined by $Prob^3$ in Equation (19), is presented by categories in Table 7.

Table 7. Average probabilities of underreporting by the model ($Prob^3$) and the magnitude of gap ($1-Prob^3$)

	Probability of underreporting	The gap
Jewellery and watches (item 18)	0.928	0.072
Furniture, including mattresses, and bed springs (item 29)	0.912	0.088
Kitchen and other household appliances (item 30)	0.930	0.070
China, glassware, tableware, and utensils (item 31)	0.909	0.091
Other durable house furnishings (item 32)	0.964	0.036
Ophthalmic products and orthopedic appliances (item 46)	0.807	0.193
Tires, tubes, accessories, and other parts (item 73)	0.874	0.126
Books and maps (item 87)	0.838	0.162
Wheel goods, sports and photographic equipment, pleasure boats and aircraft (item 90)	0.891	0.109
Video and studio products, computing equipment, and musical instruments (item 91)	0.890	0.110

As noted earlier, the probability of underreporting provides information regarding the degree of difference in what is reported as a percentage of all expenditures. Regarding the probability of underreporting by $Prob^3$, there are five clusters of items in which the gap, that is defined by the magnitude of 1.0 minus the probability of

underreporting, between microeconomic and macroeconomic statistics is less than ten percent. There are five clusters in which the gap is between ten and twenty percent.

The numbers in Table 7 are compared with those of consumer durables derived from macroeconomic statistics. The comparison of CES to PCE is reported in the *Consumer Expenditure Survey,* 1996-97 compiled by the BLS. Table 8 shows the estimates for consumer durables from the table titled "Comparisons with Other Data Sources" as presented in the *Consumer Expenditure Survey,* 1996-97 (pp. 17-22).

Table 8. Underreporting rate (Ratio of CES to PCE) (Prob[4]) and the magnitude of gap (1- Prob[4])

1994	Ratio of CES to PCE	The magnitude of gap (1-Prob[4])
Household furnishings and equipment	.66	.34
Television, radios, and sound equipment	.60	.40

Source: *Consumer Expenditure Survey,* 1996-97.

Reference table for Table 7. Ratio of CES to PCE for some categories of item from *Consumer Expenditure Survey*, 1996-97.

1994	Ratio of CES to PCE
Food, total	.73
Alcoholic beverages	.35
Household furnishings and equipment	.66
Apparel and services	.55
Transportation	.89
Entertainment	.57
Television, radios, and sound equipment	.60

Though it is difficult to compare the numbers directly because the degree of aggregation differs and there is still an issue of comparability

between the CES and PCE, we can draw some conclusions from Tables 7 and 8. From Table 8, the CES is underestimated compared to the PCE by 34 percent in household furnishings and equipment. Examining consumer durables related to household furnishings and equipment in Table 7, the categories of Furniture, including mattresses, and bed springs (item 29), Kitchen and other household appliances (item 30), China, glassware, tableware, and utensils (item 31) and, Other durable house furnishings (item 32) correspond to the category of household furnishings and equipment. For these four categories, the gap is about 7 percent on average, indicating that about 20 percent (the ratio of 7 to 34) of the gap between CES and PCE can be explained by the misreporting hypothesis.

The category of Television, radios, and sound equipment in Table 8 corresponds to that of Video and studio products, computing equipment, and musical instruments (item 91). The gap between the CES and PCE is 40 percent and the gap obtained from the present model is 11 percent, indicating that 28 percent of the gap between CES and PCE can be explained by the misreporting hypothesis.

Finally, the probability of misreporting for the item is calculated in Table 9. The probability of misreporting is obtained by $(1/n)\sum_i \Phi(-X_{2i}'\gamma)\Phi(X_{1i}'\beta/\sigma)$.

Table 9. Probability of misreporting: Prob[5]

	Probability of misreporting: Prob[5]
Jewellery and watches (item 18)	0.015
Furniture, including mattresses, and bed springs (item 29)	0.019
Kitchen and other household appliances (item 30)	0.018
China, glassware, tableware, and utensils (item 31)	0.016
Other durable house furnishings (item 32)	0.010

Ophthalmic products and orthopedic appliances (item 46)	0.034
Tires, tubes, accessories, and other parts (item 73)	0.025
Books and maps (item 87)	0.025
Wheel goods, sports and photographic equipment, pleasure boats and aircraft (item 90)	0.018
Video and studio products, computing equipment, and musical instruments (item 91)	0.034

The probability of misreporting is between 1 and 4 percent for all items, indicating that the number of households that purchased consumer durables, but did not report such expenditures was about 70 to 280 households among the sample households, depending on the items. The probability of misreporting is less than 2 percent for six items. The probability of misreporting is between 2 and 3 percent for two items. It is between 3 to 4 percent for two items.

7. Conclusion

This paper tests the misreporting hypothesis through the p_i-tobit and the extended models. The data used for the estimation involves ten clusters of consumer durables classified by the Bureau of Economic Analysis in the *National Income and Product Accounts*, Personal Consumption Expenditures as defined in the U. S. for 1994.

Regarding the extended model, the variance of the measurement error is small and is not significant statistically. This indicates that introduction of the measurement error does not improve the p_i-tobit model, and confirms that reporting of positive expenditures for consumer durables is accurate from a statistical point of view. We therefore found that the main source of underreporting is due to the existence of zero

expenditure households that in reality purchased consumer durables that were not reported in the survey.

For ten consumer durables, the probability of underreporting is less than 20 percent for all households. Compared to the results of the CES-PCE comparison by the BLS, the misreporting hypothesis plays a significant role in the underreporting observed in the micro-data sets. The present results do not contradict Slesnick [12], who found that the microeconomic data of the CES is underreported compared to the macroeconomic data of the PCE by 35 percent in 1989. Based on the CES-PCE comparison conducted by the BLS, Slesnick and the present analysis, it is clear that the gap between macroeconomic and microeconomic statistics needs to be analyzed numerically and that improvements in data collection would be useful. After we determine the estimates of the underreporting rate between CES and PCE on the same aggregation design, we can compare the probability of underreporting with the underreporting rate more accurately.

Finally, although the number of households that purchased consumer durables but did not report such expenditures in the survey is small, ranging from 1 to 4 percent of households, this factor strongly influences the degree of underreporting.

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